ABSTRACT

From a modern asset allocation perspective, risk budgeting is the mirror opposite of capital budgeting.[1] *That is, while capital budgeting is concerned with how money is distributed among asset classes, risk budgeting focuses on how risk is distributed throughout a portfolio. This paper introduces the concept of risk budgeting, and how it is applied in different portfolio construction and monitoring techniques.*

RISK BUDGETING

Thinking about asset allocation in terms of capital is natural. If an investor has $100, she may allocate $20 to stocks, $70 to bonds, and $10 to cash. Stocks will then represent ($20/$100 =) 20% of the capital in her portfolio; bonds, 70%; and cash, 10%. Through periodic portfolio rebalancing, she will maintain 20% to stocks, 70% to bonds, and 10% to cash. In short, capital (or return) budgeting makes sense to most investors.

On the other hand, thinking about asset allocation in terms of risk is relatively unnatural. While our investor has 20% of her capital in stocks (and therefore expects 20% of the portfolio's return to come from stocks), it is not obvious what proportion of the overall portfolio risk that stocks represent. To determine the proportion, she must first calculate the total amount of risk in her portfolio. Next, she must calculate her stocks' contribution to that total risk. If she is uncomfortable with the amount of risk that stocks represent, she will have to work backwards to determine the amount of money to move into or out of stocks. This line of thinking is neither natural nor easy.

Given the relative ease of adopting a capital budgeting perspective, which is concerned with how money is distributed among asset classes, it is no surprise that it is the primary approach employed by institutional investors and their advisors. However, there are several arguments for adopting, or at least incorporating, the risk budgeting perspective, which is concerned with how risk is distributed throughout a portfolio. Chief among these arguments is that a focus on risk exposure more closely conforms to financial market theory. After all, an investor does not earn a return for investing in stocks *per se*, but theoretically earns a return for assuming the various risks that may cause stocks to drop in value. For this reason, an investor should be concerned with gaining broad exposure to different market risks; that is, risk should be budgeted. Even if unconvinced by the theoretical argument, an investor should find that a risk budgeting perspective will provide additional insights into the composition of her portfolio.

A risk budgeting perspective can lead to different recommendations for portfolio construction and monitoring, some of which will be covered at the end of this brief paper. In general, these recommendations result in an allocation that is quite different from the more familiar 60/40 equity/bond split and requires a more complicated rebalancing procedure.

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[1] This use of "capital budgeting" differs slightly from the primary definition, which is concerned with how money is allocated to capital projects within a business. Capital budgeting in the context of this paper refers to how money is allocated across investments in a portfolio.

First, however, the methodologies used to determine the risk budgets of a portfolio will be discussed.

RISK BUDGETS BY INVESTMENT

Assume that an investor has a passive portfolio consisting of 60% stocks and 40% investment grade bonds by market value, as shown in the left pane of Figure 1. Using fairly standard assumptions for each asset classes' volatility[2] and correlation, the risk budget is shown graphically in Figure 2. Notice that 86% of the portfolio risk can be attributed to the stocks allocation; thus the phrase "while stocks make up 60% of the portfolio, they account for nearly 90% of the risk." Though this example covers only two asset classes, the methodology can be extended seamlessly to more typical institutional portfolios, as shown in Figure 2. As in Figure 1, the riskier assets (e.g., equities) generally assume a larger share of the overall portfolio risk than they do overall portfolio capital.

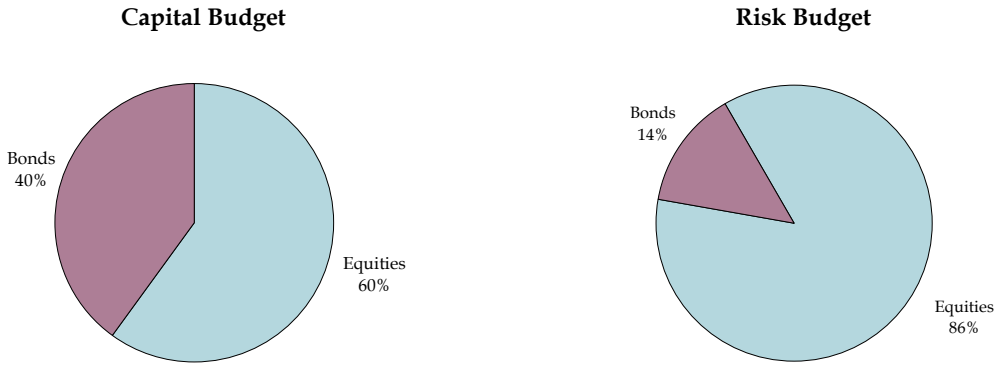

Figure 1. Capital and risk budgets of a passive portfolio composed of 60% stocks and 40% investment grade bonds

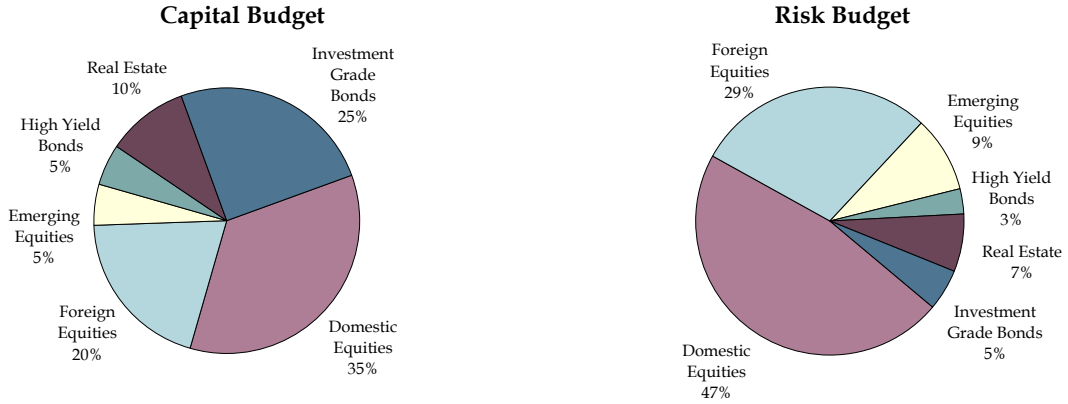

Figure 2. Capital and risk budgets of a typical institutional portfolio.

[2] In this context, the term "volatility" refers to the variance of returns. However, note that risk can assume many different forms other than the variance of returns, and risk budgeting may involve other less common measures of risk, such as value-at-risk or semi-deviation. Nevertheless, this paper focuses on variance (and standard deviation, the square root of variance) as the main measure of portfolio risk because it is the most common. The general concepts can be extended at least on a qualitative basis for the other measures of risk.

RISK BUDGETS BY FACTOR

Unlike capital budgeting, risk budgeting is not confined to investments. Instead, an investor can determine the amount of risk exposure his portfolio has to various *factors*, or abstract constructs that affect portfolio returns, such as inflation and economic growth. To assign risk budgets to factors, an investor must have established a relationship between each investment and various factors.[3] Once these relationships have been determined, the investor can envision the behavior of his portfolio as partly depending on the behavior of the factors. Then, after making some volatility and correlation assumptions among the factors, risk budgets can be generated using a complicated procedure amid some simplifying assumptions. For example, a risk budget report on the portfolio's exposure to growth and inflation might look like Figure 3.

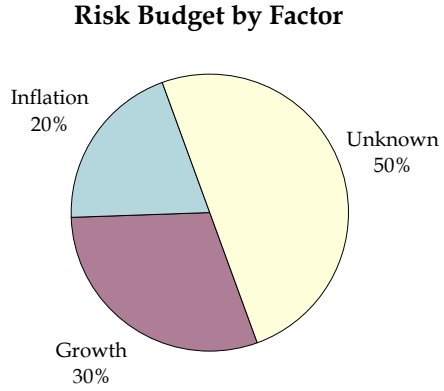

Figure 3. Risk budget by factor for hypothetical portfolio.

The hypothetical portfolio in Figure 3 is more exposed to growth risk than inflation risk; in other words, this portfolio is geared toward growth. A growth "surprise" (such as a fantastic year of productivity gains) will likely cause the portfolio's value to change more than when inflation surprises. Also, note that unknown factors account for 50% of the portfolio's risk. These unknown factors are unrelated to growth and inflation, but nonetheless influence the volatility of the portfolio. As more factors are added, the risk budget of the unknown factors will decrease.

This type of risk budgeting analysis suffers from a few practical constraints. First, it is necessary to develop a set of economic or environmental factors that are of interest to an investor. In the example above, those factors were economic growth and inflation, but the list could include more exotic factors such as the steepness of the yield curve or changes in short-term interest rates. The chosen factors should include all variables that the investor either (a) wants strategic exposure to (e.g., economic growth) or (b) wants to minimize exposure to (e.g., inflation).

Second, each investment or asset class must be related to each chosen factor, which can involve complex statistics. Furthermore, because an investor does not directly invest in factors such as "growth," but in actual assets, it is challenging to infer how a change in the amount dedicated to a particular investment will alter the risk factor picture. Instead, simple rules of

[3] This formula is usually generated from a series of linear regressions of the investments' returns on the factors. Alternatively, an investor may directly estimate a formula for his portfolio as a function of the factors.

thumb and a basic understanding of the various investment options must often suffice to guide the investor through a risk factor allocation.

The primary advantage of this type of analysis is that it enables an investor to design a portfolio that is responsive to particular strategic and hedging concerns. Furthermore, it provides a deeper insight into how a portfolio is constructed: investing is not about exposure to asset classes, but exposure to the risks and factors for which the investor hopes to be rewarded.

RISK BUDGETS BY ACTIVE OR PASSIVE

Decomposing a portfolio's risk into its active and passive components is also possible using risk budgeting techniques. This approach is particularly common, because most investors employ active managers and aim to keep active risk within reasonable limits to prevent what might be considered unacceptably high tracking error at the portfolio level.

Calculating active and passive risk budgets involves a combination of the methods previously discussed. In this case, individual portfolios are broken down into passive and active components. The passive component is usually an appropriate benchmark and behaves much the same way as a factor described above. The active component is the residual—or the "unknown"—factor. A sample passive/active risk budget picture is shown in Figure 4.

Risk Budget by Active or Passive

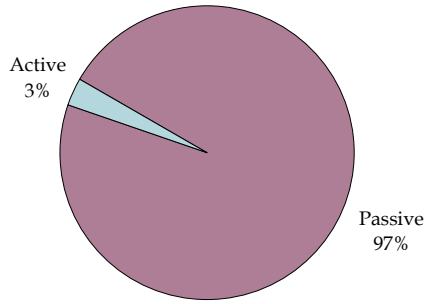

Figure 4. Risk budget by active or passive for hypothetical portfolio

The passive risk budget itself is the sum of the risk budgets of the various asset classes, while the active risk budget is the sum of the risk budgets of the various managers. In the case of Figure 4, 97% of the portfolio's risk comes from the behavior of the main asset classes in which the portfolio is invested. The remaining 3% of the portfolio's risk stems from the individual managers employed by the investor.[4]

[4] In the special case of no correlation among managers, which may occasionally be reasonable, each manager's risk budget is its share of the squared dollar-averaged standard deviations of each manager's tracking error (the variance of the residual component).

COMMON APPLICATIONS OF RISK BUDGETS

Risk budgets lend themselves to a variety of portfolio construction and monitoring applications, a few of which will be briefly discussed here.

Risk Parity

First, so-called "enhanced beta" or "risk parity" strategies essentially advocate spreading the risk budget equally across several asset classes, as a more efficient and less volatile portfolio can often be attained in this manner.[5] This can be done in one of two ways. The first involves decreasing the weight to the riskiest asset classes, thereby rendering their risk share comparable to that of the less risky asset classes. A major shortcoming of this first risk parity approach is that a shift away from risky assets implies a decrease in the overall return of the portfolio: in general, shifting away from risk assets would generate an expected portfolio return that could be unacceptably low for many institutional investors.

A second way to achieve risk parity is to leverage an allocation to a less risky asset class. For example, purchasing twice the number of bonds as an investor has money for would imply that the investor's *bond allocation has similar return and volatility characteristics to his stocks allocation*.[6] The similarity of these risk characteristics implies that a roughly equal capital allocation to stocks and to bonds would ensure that the risk budget of each was roughly half. Furthermore, the expected return of his portfolio has now increased, thereby avoiding the concerns about an unacceptably low expected return. However, by pursuing this second approach, the investor has now assumed more overall risk through his leveraged bond position.

Active Risk

"Active risk" strategies involve determining the appropriate amount of active risk to have at the portfolio level, as well as within each asset class. Unfortunately, there is no optimal level: the appropriate amount of risk is a matter of informed opinion. If an investor can tolerate substantial tracking error at the portfolio level, than increasing active risk could result in a higher total return—but only provided that talented active managers are employed in asset classes where they have a higher likelihood to add value. On the other hand, investors whose passive portfolios meet their investing needs may not feel that the potential gains from active management are worth the additional tracking error. From a practical standpoint, active risk budgeting is particularly difficult because the appropriate amount of active risk depends crucially on how capable an investor or consultant is at evaluating managers' correlations and return expectations. Given that most managers have limited histories on which to draw statistical conclusions, this task can be challenging.

[5] These strategies are often an indirect way of attempting to budget by risk factor. Since certain economic environments favor certain asset classes, a more appropriate portfolio for a highly uncertain time might be one in which a variety of asset classes share risk equally. "All Weather" strategies exemplify this approach.

[6] When an investor leverages his capital by 100%, so that for every $1 of capital he purchases $2 of assets, the return on his invested capital is now double that of the entire allocation—as is the risk. This type of financial leverage is usually attained in the futures markets.

Monitoring

Risk budgets are also used to monitor how well the portfolio is tracking its stated policy. In the first phase, standard mean-variance optimization is used to develop a policy portfolio—without regard to risk distribution. In the second stage, policy risk budgets and risk targets are determined. In the final monitoring stage, an investor continually evaluates the risk budgets of the various investments as market conditions change. A significant deviation of an investment's current risk budget from its policy risk budget could be cause for action, including rebalancing back to a risk target. This monitoring can be complex and is therefore not as common as monitoring based on capital allocations. However, it does offer the advantage of frequently incorporating new estimates of correlations and standard deviations to form a more complete picture of whether a portfolio is drifting from its policy target.[7]

CONCLUSION

Risk budgeting complements capital budgeting through its focus on how risk is allocated across investments in a portfolio. Risk budgeting's primary benefit is that it can help to illuminate how a portfolio's risk is spread across investments, factors, or active management. Armed with this knowledge, an investor can evaluate his risk exposure and make adjustments as necessary: Am I comfortable with the level of active risk in my portfolio? Am I comfortable being so highly exposed to changes in the short-term interest rate? Am I comfortable having 90% of my portfolio's risk in equities?

The primary problem of risk budgeting is its operational difficulty. Calculating risk budgets by factor or active management can be demanding—both require exhaustive data and statistical analysis. And while risk budgets by investment are relatively easy to calculate, determining how the risk budgets vary with changes in standard deviations, correlations, and amounts of various investments is challenging and unintuitive. Thus, while risk budgeting can be helpful, it is not without practical drawbacks that limit its broad acceptance.

[7] For example, if current estimates of the equity volatility have decreased from when the asset allocation policy was developed, then an increase in the dollar share of the stocks allocation may not have changed its overall risk budget. Thus, rebalancing the equity investment back to a dollar share target may not be necessary.

APPENDIX A

EXPLORING RISK BUDGETS

To become comfortable with risk budgeting, it helps to know various important facts about them. The five facts presented below are not obvious: though the math behind risk budgeting is relatively simple to state, it is complicated to analyze.

Fact 1 -- An investment's risk budget can be less than 0%.

Fact 2 -- An investment's risk budget can be greater than 100%.

These facts are easily established by setting the correlation between stocks and bonds to -0.6, while maintaining otherwise standard assumptions. In such a case, the risk budget of stocks is 113% and the risk budget of bonds is -13%. The intuition behind such a strange result is that the 40% allocation to bonds is actually responsible for a net decrease in the overall volatility of the portfolio. In essence, the diversification ability of the investment outweighs its own riskiness. The fact that stocks are greater than 100% is simply the result of it being the only other investment in the portfolio, and risk budgets must sum to 100%.[8]

Fact 3 -- As an investment's volatility changes within a historically normal range, its risk budget may either increase or decrease. However, as an investment's volatility becomes extremely large, its risk budget approaches 100%.

The first part of this fact simply states that an increase in an investment's volatility does not necessarily translate to an increase in its risk budget—a relationship that might seem natural *but is not true in many cases*. The second part of this fact is particularly intuitive: as an investment's volatility becomes extremely large, it makes sense that it dominates the overall portfolio risk.

Fact 4 -- As an investment's capital share changes, its risk budget may either increase or decrease. However, as an investment's capital share goes to 100%, its risk budget approaches 100%.

The first part of this fact states that an increase in the amount of money in an investment's does not necessarily translate to an increase in its risk budget—a relationship that also might seem natural *but is not true in many cases*. The second part of this fact is straightforward: as an investment dominates the portfolio in dollar terms, it makes sense that its risk budget approaches 100%.

Fact 5 -- As correlations among investments change, risk budgets will change in various ways. However,

1. As correlations among all investments approach one, the risk budget of any investment will be its share of the dollar-averaged standard deviations of the portfolio's investments.
2. As correlations among all investments approach zero, the risk budget of any investment will be its share of the squared dollar-averaged standard deviations of the portfolio's investments.

[8] If one investment among many has a negative risk budget, then the sum of all the remaining investments' risk budgets will be greater than 100%.

Similar to Facts 3 and 4, the first statement of Fact 5 notes that it is difficult to assess how risk budgets change as the correlations among the investments change. However, in the special cases when all correlations are or go to one (as often happens during a crisis) and when all correlations are or go to zero (as might happen in an exceptionally diversified portfolio), the risk budgets follow a simple rule. In the prior example, if the correlation between stocks and bonds happened to be one, then the risk budgets would be 75% and 25%, respectively.[9] If the correlation between stocks and bonds happened to be zero, then the risk budgets would be 90% and 10%, respectively.[10] Paradoxically, in this example stocks represent a smaller share of the risk during a crisis than during more normal and "diversifiable" times.

[9] When the correlation between bonds and stocks is one, then the risk budget for stocks is 0.60*0.16/(0.60*0.16 + 0.40*0.08), which equals 75%. The risk budget for bonds is 0.40*0.08/(0.60*0.16 + 0.40*0.08), which equals 25%.

[10] When the correlation between stocks and bonds is zero, then the risk budget for stocks is $(0.60*0.16)^2/((0.60*0.16)^2 + (0.40*0.08)^2)$, which equals 90%. The risk budget for bonds is $(0.40*0.08)^2/((0.60*0.16)^2 + (0.40*0.08)^2)$, which equals 10%.

REFERENCES

Rahl, Leslie, ed. *Risk Budgeting: A New Approach to Investing*. London: Risk Books, 2000.

Sharpe, William. "Budgeting and Monitoring the Risk of Defined Benefit Pension Plans." Published Lecture Notes. September, 2001.